Critical Values for IPS Panel Unit Root Tests: A Response Surface Analysis

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By

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Abstract

This paper presents the critical values for the testing of unit roots in heterogeneous panels. The paper develops an algorithm to generate the critical values through Monte Carlo simulations which will be computationally efficient as opposed to the traditional simulation techniques used in the earlier panel unit root studies. The results from the simulation experiments are used to construct the response surface regressions in which the critical values depend on both cross-sectional and time units. The predictability of the response surface regressions are evaluated through reported IPS critical values.

Key words: Panel unit root tests, response surface regressions, randomization, t-bar.

1. Introduction

The use of panel unit root tests has become very popular among applied econometricians since the development of panel unit root test procedures by Levin and Lin (1992, 1993).[1] One of the advantages of this procedure is that the power of the test increases with an increase in the number of panel series compared to the well-known low power of the standard ADF unit root test against near unit root alternatives.[2] Increasingly, recent empirical studies use the test procedure introduced by Im, Pesaran and Shin (2003) (hereafter, IPS) which can test the null hypothesis of non-stationarity in the presence of heterogeneity across the panel. Most of the empirical literature uses either the critical values reported by IPS which are close to their sample sizes or Monte Carlo experiments for their particular sample sizes.[3] On the other hand, other researchers use standardized t-bar test statistics (see section 2) to verify the panel unit root properties of the data.[4]

The inferences based on IPS critical values could be misleading when the sample size is approximated to the reported values. In this paper, we propose an algorithm to obtain the critical values for non-standardized t-bar statistic without conducting an extensive simulation for the individual cross-sections. The critical values obtained from

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[1] The Augmented Dickey-Fuller (ADF) test for stationarity has been extended to panel tests for stationarity under models with various degrees of heterogeneity by, for example Levin and Lin (1992,1993), Quah (1994) and Im, Pesaran and Shin (2003) (hereafter, IPS). The main difference between the panel unit test procedures proposed by Quah (1994), Levin and Lin (1992) and IPS is that while the former construct the test statistic under the alternative hypothesis that all component series in the panel are stationary, the latter (IPS) test the alternative that at least one of the individual series is stationary.

[2]These panel unit root tests have been employed in various studies to verify the validity of the various hypotheses and the economic theories. To cite a few, for example Purchasing Power Parity hypotheses (see, MacDonald (1996), Oh (1996), Wu (1996), Frankel and Ross (1996), Coakley and Fuertes (1997, 2000) Papel (1997) and Fleissig and Strauss (2000)), panel unit root properties of inflation rate (see Culver and Papel (1997), Lee and Wu (2001) and Holmes(2002)), unit roots in health care expenditure and GDP (see, McCoskey and Selden (1998)), Investment-Saving correlation (see, Ho (2002)), Mean reversion of interest rates (see, Wu and Chen (2001)), Gibrat's Law of Proportinate effects (see,Goddard, Wilson, Blandon (2002)).

[3] See Ho (2002), Holmes (2002), Wu and Chen (2001), Goddard, Wilson and Blandon (2002),Cushman, MacDonald and Samborsky (2001), Chou and Chao (2001), Holmes (2001) and Strauss (2000).

[4] See, for example, Fleissing and Strauss (2000), Strazicich, Co and Lee (2001) and Wu (2000)

these experiments are summarized by means of response surface regressions in which the critical values depend on the sample size (see MacKinnon (1991)). The predictability of the response surface regressions are evaluated by comparing the predicted critical values with reported IPS critical values. Both in-sample and out-of-sample predictability of the regressions are evaluated through the errormetrics such as root mean squared error (RMSE), mean absolute error (MSE) and mean absolute percentage error (MAPE). Finally, the paper reports the critical values based on the estimated response surface regression for the IPS sample.

2. IPS Panel Unit Root Test

The heterogeneous panel data model proposed by IPS is given by

$$\Delta y_{it} = \mu_i + \beta_i y_{it-1} + \sum_{k=1}^{p_i} \varphi_k \Delta y_{it-k} + \gamma_i t + \varepsilon_{it}, \; i=1,2,\ldots,N, \; t=1,2,\ldots,T. \qquad (1)$$

The null and alternative hypotheses are $H_0 : \beta_i = 0, \quad H_1 : \exists i \quad st \; \beta_i < 0$. Each equation is estimated separately by OLS due to heterogeneity and the test statistics are obtained as (studentized) averages of the test statistics for each equation.

The t-bar statistic proposed by IPS is defined as the average of the individual Dickey-Fuller τ statistics: $\bar{t} = \frac{1}{N}\sum_{i=1}^{N}\tau_i$, where $\tau_i = \frac{\hat{\beta}_i}{\hat{\sigma}_{\beta_i}}$. (2)

IPS report the critical values for the t-bar statistics described by (2) for the various combinations of *N* and *T*.

The standardized t-bar statistic proposed by IPS under the assumption that the cross-sections are independent is given by

$$\Gamma_i = \frac{\sqrt{N}\left(\bar{t} - E(\tau_i \mid \beta_i = 0)\right)}{\sqrt{\operatorname{var}(\tau_i \mid \beta_i = 0)}}. \tag{3}$$

The means $E(\tau_i \mid \beta_i = 0)$ and the variances $\operatorname{var}(\tau_i \mid \beta_i = 0)$ are obtained by Monte Carlo simulations and are tabulated in IPS. IPS conjecture that the standardized t-bar statistic Γ_i converges weakly to a standard normal distribution as *N* and $T \to \infty$.

3. Simulation Experiments

The underlying data generating process (DGP) considered by IPS is $y_{it} = y_{it-1} + \varepsilon_{it}$, $\varepsilon_{it} \sim N(0,1)$, $t = 1,2,...,T$; $i = 1,2,...,N$, with $y_{i0} = 0$. They estimate t-bar statistics based on (1). The critical values reported by IPS are computed via stochastic simulation of 50,000 replications for the models with 1) a constant and 2) a constant and a trend. In this paper, we estimate the response surface function to approximate the lower-tail critical values of 1 percent, 5 percent and 10 percent for the models with 1) a constant and no trend and 2) a constant and a trend. The simulation technique introduced in this paper is different from the usual Monte Carlo experiments adopted by IPS. Instead of simulating the underlying DGP and re-estimating the model (1) for the various combinations of *N* across the *T*, this paper randomizes the *t*-statistic across the replications obtained from a model with a single cross-section for a fixed sample size, *T*. We use *M*=100,000 replications for this purpose. On the other hand the simulation experiment is conducted for the sample of *N*=1 with *T* observations only.

3.1 Algorithm

The underlying data generating process in the simulations is given by $y_t = y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$, t = 1, 2,…,*T*. In the first stage, the underlying DGP is generated and the ADF regression is fitted for the simulated data of size *T* over *M* replications. It should be noted that the underlying DGP is not generated for the panel of size N as in the traditional approaches. The t-statistic to test the null hypothesis of $\beta = 0$ in (1) is computed for a single cross-section of size T over M replications.

Let $t_{11}, t_{12}, ..., t_{1M}$ be the corresponding estimated t-statistic for the first cross-section over M replications. Secondly, the t-statistics over M replications for the remaining N-1 cross-sections can be obtained by simply randomizing the first M t-statistics obtained from the cross-section of size 1 (i.e., $t_{11}, t_{12}, ..., t_{1M}$). That is, the t-statistics t_{ij}, $i = 2,3,...N$; $j = 1,2,..., M$ are constructed by $t_{ij} = t_{1[k]}$, where the replication index k is randomly drawn from a uniform distribution by a simple random sampling with replacement[5] (i.e. $k \sim U[1, M]$). Here [k] refers to the integer part of the given argument k. The cumulative averages over the N cross-sections, $\frac{1}{n}\sum_{i=1}^{n} t_{mi}$; $n = 1,2,..., N$, constitute the t-bar statistics for the m-th replication. Finally, the critical values are obtained by extracting the 1st, 5th and 10th quintiles from the simulated numerical distribution.[6] It is observed that during the simulations the proposed algorithm presents the same critical values as the traditional Monte Carlo simulation technique. The proposed simulation mechanism is tabulated in Appendix 1.

Using this algorithm, one can obtain the critical values for $n = 1,2,...N$ for the fixed sample size T through cumulative averages. However, the traditional simulation approaches are able to provide the critical values for fixed N and T. For a fixed T, only M

[5] The results obtained from simple random sampling with replacement (SRSWR) are consistent with the simple random sampling without replacement (SRSWOR) as the number of replications M=100,000 are sufficiently large.

[6] In this exercise, we develop the response surface regression for non-standardized test statistics as the standardized test statistics involves larger number of parameters that has to be estimated by stochastic simulations. Therefore, the sampling error of estimating non-standardized test statistics will be smaller than that of standardized ones.

experiments need to be conducted using the proposed algorithm as opposed to the traditional approaches that require *NM* experiments to obtain the desired critical values. The cost of computing the remaining (*N-1*)*M* relevant test statistics by randomization is significantly less than that of the traditional one. The computational time for the traditional approaches increases significantly as *T* increases. It is expected the new algorithm will provide new insights for panel regression studies because it is computationally efficient.

4. Response Surface Analysis

In order to generalize the estimators of the critical values for any combination of cross-sectional unit *N* and the sample size *T* at a given level of significance, we use the response surface regression techniques proposed by MacKinnon (1991, 1996). Suppose that we are interested in $q_{\bar{t}}^{\alpha}(T,N)$, i.e., α quantile of the distribution, where α = 1%, 5% and 10%. Response surfaces are estimated for two different tests[7]: 1) *t*-bar statistic with a constant 2) *t*-bar statistic with a constant and a trend. In each case, three response surfaces are estimated based on the 1st, 5th and 10th quantiles. Hence, a total of six response surface regressions are estimated. We consider all combinations of $N \in \{1,2,\ldots,100\}$ and $T \in \{5,6,7,\ldots,100\}$. The number of observations used in each response surface regression is 9600.

In contrast to response surface regressions based on pure time series studies, in which the regression equation is a function of sample size *T*, we construct the response surfaces equation which is a function of *T* and *N* and the response surface equation for the *t*-bar test statistic:

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[7] See MacKinnon (1996), MacKinnon, Haug and Michelis (1999) for more details about response surface regression techniques.

$$
\begin{aligned}
q_{\bar{t}}^{\alpha}(T,N) = {} & \beta_0 + \beta_1 N^{-1} + \beta_2 N^{-2} + \beta_3 N^{-3} + \beta_4 T^{-1} + \beta_5 T^{-2} + \beta_6 T^{-3} + \\
& \beta_7\left(\frac{N}{N+1}\right) + \beta_8\left(\frac{N}{N+1}\right)^2 + \beta_9\left(\frac{T}{T+1}\right) + \beta_{10}\left(\frac{T}{T+1}\right)^2 + \beta_{11}N^{-1}T^{-1} + \beta_{12}N^{-1}T^{-2} + \beta_{13}N^{-1}T^{-3} + \\
& \beta_{14}N^{-2}T^{-1} + \beta_{15}N^{-2}T^{-2} + \beta_{16}N^{-2}T^{-3} + \beta_{17}N^{-3}T^{-1} + \beta_{18}N^{-3}T^{-2} + \beta_{19}N^{-3}T^{-3} + \\
& \beta_{20}N^{-1}\left(\frac{T}{T+1}\right) + \beta_{21}N^{-2}\left(\frac{T}{T+1}\right) + \beta_{22}N^{-3}\left(\frac{T}{T+1}\right) + \\
& \beta_{23}T^{-1}\left(\frac{N}{N+1}\right) + \beta_{24}T^{-2}\left(\frac{N}{N+1}\right) + \beta_{25}T^{-3}\left(\frac{N}{N+1}\right)
\end{aligned}
\tag{4}
$$

In the response surface equations, the regressors are chosen to minimize the root mean squared error of the regression. The regressors T^{-k}'s and N^{-k}'s capture the individual time and cross-sectional effects respectively. It is observed that for a fixed *T*, the critical value $q_{\bar{t}}^{\alpha}(N,T)$ is an increasing function of *N* and vice versa. The regressors N^{-k} and T^{-k} do not explain such effects completely. In order to capture such monotonicity and to ensure the convergence of the response surface regressions for large *N* and *T*, we introduce $\left(\frac{N}{N+1}\right)^k$ and $\left(\frac{T}{T+1}\right)^k$ as additional explanatory variables. It is found during the experiments that the response surface equation with these factors outperforms the models without these factors. Furthermore, the response surface equation is improved by multiplying these factors by N^{-k} and T^{-k}. These multiplicative terms then incorporate the effects from the interaction of *N* and *T*. It is also observed that the critical values are more sensitive to *T* when *N* is small than when it is large. These effects are also captured through the interaction of N^{-k} and T^{-k} with the factors $\left(\frac{N}{N+1}\right)$ and

$\left(\frac{T}{T+1}\right)$. It is also observed that the inclusion of such interaction factors for the higher degree, for example $\left(\frac{T}{T+1}\right)^2$, does not improve the results.

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Tables 1 and 2

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The performance of the response surface regressions are evaluated by both within-sample and out-of-sample predictability of the critical values. The response surface regressions are chosen to minimize the root mean squared error (RMSE) of the regressions. For the out-of-sample predictions, we conduct Monte Carlo experiments for the combinations of $N \in \{1,2,3,...,N\}$ and $T \in \{200, 300, 400, 500\}$. This constitutes 400 samples for each case. This helps to evaluate the accuracy of the response surfaces for large *T*. Three measurements - root mean squared errors (RMSE), mean absolute errors (MAE), mean absolute percentage errors (MAPE) are used to evaluate the performance of the estimated response surface regressions for *t*-bar test statistics. The results are reported in Table 3. The predictability of the estimated response surface equation is also compared with reported critical values from the IPS study. It is also observed for the models with a constant and a trend that the reported critical values for *T*=5 in the IPS paper for 50,000 replications are quite different from the critical values generated (by Monte Carlo simulation) in this paper based on 100,000 replications. These discrepancies could be due to the significant difference in the number of replications. It is necessary to have a large number of replications for the case of *T*=5 because individual Dickey Fuller regression suffer from a lack of degrees of freedom for the models with the constant and the trend because three parameters with a sample of 5 are estimated. We have also verified the accuracy of our critical values by adopting 200,000 replications and the critical values are same as for 100,000 replications. The error-metrics for the IPS sample by excluding *T*=5 are also reported in Table 3. Finally, the critical values for the IPS sample based on estimated response surface functions are reported in Table 4. The estimated response surface regressions are portrayed in appendix 2.

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Table 3 and 4

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It is observed from table 3 that the response surface regressions provide smooth and accurate critical values at 3 decimal places and the average predictive error of these regressions are less than half a percent in most of the cases. The performance of the response surface regression for the 10 percent critical values is notably better than that of the response surface regressions for the 5 percent and 1 percent critical values. The performance of the models for the 5 percent critical value is superior to the models with the 1 percent critical values. In general, the estimated models reported in Tables 1 and 2 outperform the other competitive models based on three criteria: RMSE, MAE and

MAPE.[8] For the sake of brevity, the response surface regression results for the other competitive models are not reported.

5. Conclusion

The response surface regressions for the IPS critical values should prove to be useful for applied econometricians testing unit roots in heterogeneous panels. The proposed algorithm to generate the critical values provides a new dimension to panel studies because it is computationally efficient and powerful. The response surface regressions were developed based on the critical values obtained from simulation experiments and are functions of the number of cross-sections and sample sizes. The performance of these regressions was compared with reported IPS critical values. The critical values for the panel unit roots of any combination of cross sections and sample sizes can be calculated in a spreadsheet by substituting the panel dimensions in the response surface regressions without conducting an extensive simulation.

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[8] We have also considered various competitive models by allowing non-linearity such as $N^{-\gamma}$ and $T^{-\delta}$, where γ and δ could non-integers, in the response surface regressions.

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Table 1: Response Surface Regressions for the t-bar statistics: Constant but no Trend

	1 percent		5 percent		10 percent	
	Coefficient	S.E	Coefficient	S.E	Coefficient	S.E
β_0	1733.20	38.43	1203.59	18.29	922.475	15.75
β_1	3570.36	1120.00	-607.08	5.28	-711.909	29.42
β_2	9914.04	4166.00	291.32	2.66	228.53	2.249
β_3	-13483.40	3194.00	-72.85	0.71	-57.2637	0.59
β_4	-626.47	69.05	-405.65	45.41	-335.719	15.07
β_5	2597.66	819.90	1343.80	593.60	180.797	13.7
β_6	-16488.00	2962.00	-9696.18	2146.00	-163.203	9.926
β_7	-1829.31	14.91	-1295.45	10.80	-1013.18	9.25
β_8	490.09	3.83	347.14	2.78	271.171	2.38
β_9	-395.63	36.80	-256.90	16.43	-182.064	14.17
β_{10}						
β_{11}	-4216.04	1120.00	136.067	41.42	380.094	29.12
β_{12}	2276.60	1348.00	-1036.25	580.4	-217.614	22.61
β_{13}	11370.00	3002.00	8903.58	2099		
β_{14}	-9649.61	4161.00	-101.755	32.97	-107.817	4.293
β_{15}	10700.70	4082.00	783.968	462	-45.8504	21.12
β_{16}	-17256.20	3727.00	-6610.15	1671	474.218	76.31
β_{17}	13423.30	3190.00	36.8017	12.74	39.0583	1.935
β_{18}	-13519.60	3101.00	-290.383	178.6	32.9965	16.24
β_{19}	13244.20	2415.00	2390.17	645.7	-310.413	58.69
β_{20}	-4427.81	1120.00			236.467	29.07
β_{21}	-9502.65	4166.00				
β_{22}	13380.60	3194.00				
β_{23}	230.98	58.48	148.901	42.36	153.816	5.169
β_{24}	-2207.43	819.40	-1090.07	593.6		
β_{25}	16159.30	2963.00	9477.73	2146		
R^2	0.999541		0.999442		0.99929	

Table 2: Response Surface Regressions for the t-bar statistics: Constant and Trend

	1 percent		5 percent		10 percent	
	Coefficient	S.E	Coefficient	S.E	Coefficient	S.E
β_0	395236.0	3409.0	155673.0	1855.0	105500.0	1475.0
β_1	114570.0	1737.0	29797.9	945.1	10340.0	509.4
β_2	-214057.0	6464.0	-67269.1	3517.0	-33584.3	1730.0
β_3	124791.0	4956.0	39446.8	2696.0	21526.0	1349.0
β_4	-287267.0	2460.0	-113168.0	1338.0	-76648.5	1064.0
β_5	203071.0	1974.0	81189.0	1074.0	53570.5	780.6
β_6	-211064.0	4636.0	-90158.2	2522.0	-54056.5	1578.0
β_7	-1812.1	23.1	-1258.9	12.6	-971.6	10.0
β_8	480.5	5.9	334.9	3.2	259.0	2.6
β_9	-501959.0	4360.0	-197021.0	2372.0	-133329.0	1886.0
β_{10}	108053.0	951.2	42270.4	517.5	28539.5	411.5
β_{11}	-114010.0	1737.0	-29716.8	945.2	-10410.8	486.3
β_{12}	88855.2	2091.0	19161.0	1138.0	4901.1	276.2
β_{13}	43462.6	4658.0	35409.5	2534.0	24581.2	1238.0
β_{14}	213289.0	6456.0	67004.0	3513.0	33487.7	1707.0
β_{15}	-191164.0	6334.0	-57847.6	3446.0	-28594.9	1347.0
β_{16}	61147.8	5782.0	5953.6	3146.0		
β_{17}	-124424.0	4950.0	-39303.4	2693.0	-21455.9	1339.0
β_{18}	115172.0	4812.0	35556.6	2618.0	19396.7	1182.0
β_{19}	-56571.8	3747.0	-13030.0	2038.0	-6641.3	503.9
β_{20}	-115429.0	1737.0	-30392.4	945.1	-10797.8	509.7
β_{21}	214475.0	6464.0	67557.1	3517.0	33805.5	1730.0
β_{22}	-124898.0	4956.0	-39519.8	2696.0	-21581.8	1349.0
β_{23}	1406.9	90.7	684.9	49.4	397.4	32.6
β_{24}	-25011.1	1271.0	-10885.1	691.7	-5800.5	436.8
β_{25}	136655.0	4597.0	60508.7	2501.0	33786.1	1564.0
R^2	0.999399		0.999421		0.999303	

Table 3: Predictability of Response Surface Regressions: t-bar test statistic

		Constant but no trend			Constant and trend		
		1%	5%	10%	1%	5%	10%
Within Sample	RMSE	0.006	0.004	0.004	0.009	0.005	0.004
	MAE	0.004	0.003	0.003	0.006	0.004	0.003
	MAPE	0.24%	0.19%	0.17%	0.22%	0.15%	0.13%
Out sample	RMSE	0.005	0.004	0.003	0.02	0.01	0.008
	MAE	0.004	0.003	0.003	0.02	0.01	0.009
	MAPE	0.22%	0.18%	0.17%	0.97%	0.44%	0.34%
IPS Reported values	RMSE	0.01	0.006	0.005	0.07	0.01	0.009
	MAE	0.006	0.004	0.004	0.02	0.007	0.005
	MAPE	0.31%	0.24%	0.24%	0.63%	0.26%	0.20%
IPS* Reported values	RMSE				0.008	0.004	0.004
	MAE				0.006	0.003	0.003
	MAPE				0.23%	0.15%	0.15%

* Comparison with IPS critical values by excluding *T*=5 case

Table 4 Critical Values of the t-bar statistic based on Response Surface Regressions

N\T	5	10	15	20	25	30	40	50	60	70	100
Panel A: DF regressions containing only constants											
1 percent											
5	-3.82	-2.66	-2.53	-2.48	-2.46	-2.44	-2.42	-2.41	-2.40	-2.40	-2.39
7	-3.45	-2.48	-2.38	-2.34	-2.32	-2.31	-2.29	-2.28	-2.28	-2.27	-2.27
10	-3.11	-2.32	-2.24	-2.21	-2.20	-2.19	-2.18	-2.17	-2.16	-2.16	-2.16
15	-2.79	-2.16	-2.11	-2.08	-2.07	-2.07	-2.06	-2.05	-2.05	-2.05	-2.04
20	-2.60	-2.07	-2.02	-2.00	-2.00	-1.99	-1.98	-1.98	-1.98	-1.98	-1.97
25	-2.47	-2.00	-1.96	-1.95	-1.94	-1.94	-1.93	-1.93	-1.93	-1.93	-1.92
50	-2.18	-1.85	-1.83	-1.82	-1.82	-1.81	-1.81	-1.81	-1.81	-1.81	-1.80
100	-2.01	-1.76	-1.75	-1.74	-1.74	-1.74	-1.74	-1.74	-1.74	-1.73	-1.73
5 percent											
5	-2.75	-2.28	-2.21	-2.19	-2.18	-2.17	-2.16	-2.15	-2.15	-2.14	-2.14
7	-2.59	-2.16	-2.11	-2.09	-2.08	-2.07	-2.07	-2.06	-2.06	-2.06	-2.05
10	-2.44	-2.06	-2.02	-2.00	-2.00	-1.99	-1.98	-1.98	-1.98	-1.98	-1.97
15	-2.28	-1.96	-1.93	-1.92	-1.91	-1.91	-1.90	-1.90	-1.90	-1.90	-1.89
20	-2.18	-1.90	-1.87	-1.86	-1.86	-1.85	-1.85	-1.85	-1.85	-1.85	-1.84
25	-2.11	-1.85	-1.83	-1.82	-1.82	-1.82	-1.81	-1.81	-1.81	-1.81	-1.81
50	-1.95	-1.75	-1.74	-1.73	-1.73	-1.73	-1.73	-1.73	-1.73	-1.73	-1.72
100	-1.85	-1.69	-1.68	-1.68	-1.68	-1.68	-1.67	-1.67	-1.67	-1.67	-1.67
10 percent											
5	-2.39	-2.09	-2.05	-2.04	-2.03	-2.02	-2.02	-2.01	-2.01	-2.01	-2.01
7	-2.28	-2.01	-1.98	-1.96	-1.96	-1.95	-1.95	-1.94	-1.94	-1.94	-1.94
10	-2.18	-1.93	-1.91	-1.90	-1.89	-1.89	-1.88	-1.88	-1.88	-1.88	-1.88
15	-2.07	-1.86	-1.84	-1.83	-1.82	-1.82	-1.82	-1.82	-1.82	-1.82	-1.81
20	-2.00	-1.81	-1.79	-1.79	-1.78	-1.78	-1.78	-1.78	-1.78	-1.78	-1.77
25	-1.95	-1.77	-1.76	-1.76	-1.75	-1.75	-1.75	-1.75	-1.75	-1.75	-1.75
50	-1.84	-1.69	-1.69	-1.68	-1.68	-1.68	-1.68	-1.68	-1.68	-1.68	-1.68
100	-1.78	-1.65	-1.64	-1.64	-1.64	-1.64	-1.64	-1.64	-1.64	-1.64	-1.64
Panel A: DF regressions containing constants and linear trends											
1 percent											
5	-7.93	-3.41	-3.22	-3.13	-3.08	-3.05	-3.02	-3.00	-2.99	-2.99	-2.98
7	-7.16	-3.20	-3.05	-2.98	-2.94	-2.92	-2.89	-2.88	-2.87	-2.87	-2.86
10	-6.39	-3.02	-2.90	-2.85	-2.82	-2.80	-2.78	-2.77	-2.77	-2.76	-2.75
15	-5.62	-2.86	-2.76	-2.72	-2.70	-2.69	-2.67	-2.67	-2.66	-2.66	-2.65
20	-5.15	-2.76	-2.67	-2.64	-2.62	-2.61	-2.60	-2.60	-2.59	-2.59	-2.58
25	-4.84	-2.69	-2.61	-2.58	-2.57	-2.56	-2.56	-2.55	-2.55	-2.54	-2.53
50	-4.15	-2.53	-2.47	-2.45	-2.45	-2.44	-2.44	-2.44	-2.44	-2.43	-2.42
100	-3.76	-2.44	-2.39	-2.38	-2.37	-2.37	-2.37	-2.37	-2.37	-2.37	-2.36
5 percent											
5	-4.62	-2.97	-2.87	-2.82	-2.79	-2.78	-2.76	-2.75	-2.75	-2.74	-2.74
7	-4.39	-2.85	-2.76	-2.72	-2.70	-2.69	-2.68	-2.67	-2.66	-2.66	-2.65
10	-4.13	-2.74	-2.67	-2.64	-2.62	-2.61	-2.60	-2.60	-2.59	-2.59	-2.58
15	-3.85	-2.63	-2.57	-2.55	-2.54	-2.53	-2.53	-2.52	-2.52	-2.51	-2.51
20	-3.66	-2.57	-2.51	-2.50	-2.49	-2.48	-2.48	-2.47	-2.47	-2.47	-2.46
25	-3.54	-2.52	-2.47	-2.46	-2.45	-2.45	-2.44	-2.44	-2.44	-2.43	-2.43

50	-3.25	-2.41	-2.38	-2.37	-2.37	-2.36	-2.36	-2.36	-2.36	-2.36	-2.35
100	-3.08	-2.35	-2.32	-2.32	-2.31	-2.31	-2.31	-2.31	-2.31	-2.31	-2.31
10 percent											
5	-3.72	-2.77	-2.70	-2.67	-2.65	-2.64	-2.63	-2.62	-2.62	-2.62	-2.61
7	-3.61	-2.68	-2.62	-2.59	-2.58	-2.57	-2.56	-2.56	-2.56	-2.55	-2.55
10	-3.48	-2.60	-2.55	-2.53	-2.52	-2.51	-2.51	-2.50	-2.50	-2.50	-2.49
15	-3.32	-2.52	-2.48	-2.46	-2.46	-2.45	-2.45	-2.44	-2.44	-2.44	-2.44
20	-3.22	-2.47	-2.43	-2.42	-2.42	-2.41	-2.41	-2.41	-2.40	-2.40	-2.40
25	-3.15	-2.44	-2.40	-2.39	-2.39	-2.39	-2.38	-2.38	-2.38	-2.38	-2.37
50	-2.98	-2.36	-2.33	-2.33	-2.32	-2.32	-2.32	-2.32	-2.32	-2.32	-2.31
100	-2.88	-2.31	-2.29	-2.28	-2.28	-2.28	-2.28	-2.28	-2.28	-2.28	-2.28

Appendix 1: Simulation Mechanism for Panel Unit Root Test

	t-statistics						*t-bar* statistics					
Replications	Cross-section1	Cross-section2	Cross-section3	.	.	Cross-section*N*	*N*=1	*N*=2	*N*=3	.	.	*N*=*N*
1	t_{11}	t_{21}	t_{31}			t_{N1}	$\bar{t}_1 = t_{11}$	$\bar{t}_1 = \sum_{i=1}^{2} t_{i1}$	$\bar{t}_1 = \sum_{i=1}^{3} t_{i1}$			$\bar{t}_1 = \sum_{i=1}^{N} t_{i1}$
2	t_{12}	t_{22}	t_{32}			t_{N2}	$\bar{t}_2 = t_{12}$	$\bar{t}_2 = \sum_{i=1}^{2} t_{j2}$	$\bar{t}_2 = \sum_{i=1}^{3} t_{j2}$			$\bar{t}_2 = \sum_{i=1}^{N} t_{j2}$
3	t_{13}	t_{23}	t_{33}			t_{N3}	$\bar{t}_3 = t_{13}$	$\bar{t}_3 = \sum_{i=1}^{2} t_{i3}$	$\bar{t}_3 = \sum_{i=1}^{3} t_{i3}$			$\bar{t}_3 = \sum_{i=1}^{N} t_{i3}$
.	.	.	.			.	.	.	.			.
.	.	.	.			.	.	.	.			.
j	t_{1j}	t_{2j}	t_{3j}			t_{Nj}	$\bar{t}_j = t_{1j}$	$\bar{t}_j = \sum_{i=1}^{2} t_{ij}$	$\bar{t}_j = \sum_{i=1}^{3} t_{ij}$			$\bar{t}_j = \sum_{i=1}^{N} t_{ij}$
.	.	.	.			.	.	.	.			.
.	.	.	.			.	.	.	.			.
M	t_{1M}	t_{2M}	t_{3M}			t_{NM}	$\bar{t}_M = t_{1M}$	$\bar{t}_M = \sum_{i=1}^{2} t_{iM}$	$\bar{t}_M = \sum_{i=1}^{3} t_{iM}$			$\bar{t}_M = \sum_{i=1}^{N} t_{iM}$

Simulation Note:

(1) *T* is fixed.

(2) Column 2 (for cross-section 1) is obtained by stochastic simulation of *M* replications based on equation (1).

(3) Values in Columns 3 through *N*+1 (i.e., cross-sections 2 through *N*)are obtained by randomly drawing the values from column 2 with replacement.

Appendix 2: Response surface function